**Appendix 2: Curriculum Connections Across the Grades: Primary**

Overview of the mathematical knowledge and skills that students are expected to acquire in the strand ***Patterning and Algebra: (Notice that some specific expectations related to Algebraic Reasoning but connected to other strands are included)***

**Grade 1:**

* creating and extending repeating patterns involving one attribute;
* introducing the concept of equality using only concrete materials

**Grade 2:**

* identifying and describing repeating patterns and growing and shrinking patterns;
* developing the concept of equality using the addition and subtraction of numbers to 18 and the equal sign;
* using the commutative property and the property of zero in addition to facilitate computation

**Grade 3:**

* creating and extending growing and shrinking patterns;
* representing geometric patterns with a number sequence, a number line, and a bar graph;
* determining the missing numbers in equations involving addition and subtraction of one- and two-digit numbers;
* investigating the properties of zero and one in multiplication

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| **Kindergarten** | | **Grade 1** | **Grade 2** | **Grade 3** |
| * investigate some concepts of quantity through identifying and comparing sets with more, fewer, or the same number of objects * begin to make use of one-to-one correspondence in counting objects and matching groups of objects * demonstrate understanding of the counting concepts of stable order * recognize some quantities without having to count, using a variety of tools * begin to use information to estimate the number in a small set * use, read, and represent whole numbers to 10 in a variety of meaningful contexts * investigate and develop strategies for composing and decomposing quantities to 10 * investigate addition and subtraction in everyday activities through the use of manipulatives * compare and order two or more objects according to an appropriate measure and use measurement terms * identify, create, reproduce, and extend repeating patterns through investigation, using a variety of materials and actions * identify and describe informally the repeating nature of patterns in everyday contexts, using oral expressionsand gestures * sort, classify, and compare objects and describe the attributes used * collect objects and data and make representations of their observations, using concrete graphs | * identify, describe, and extend, through   investigation, geometric repeating patterns  involving one attribute   * identify and extend, through investigation,   numeric repeating  patterns   * describe numeric repeating patterns in a   hundreds chart;   * identify a rule for a repeating pattern * create a repeating pattern involving one attribute * represent a given repeating pattern in a   variety of ways   * create a set in which the number of objects is greater than, less than, or equal to the number of objects in a given set; * demonstrate examples of equality, through   investigation, using a  “balance” model   * determine, through investigation using a   “balance” model and  whole numbers to 10, the  number of identical  objects that must be  added or subtracted to  establish equality   * solve a variety of problems involving the   addition and subtraction  of whole numbers to 20,  using concrete materials  and drawings (Number  Sense and Numeration)   * solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (Number Sense and Numeration) * pose and answer questions about collected data (Data Management and Probability) | | * identify and describe, through investigation,   growing patterns and shrinking patterns generated by the repeated addition  or subtraction of 1’s, 2’s, 5’s, 10’s, and 25’s on a number line and on a hundreds chart   * identify, describe, and create, through investigation, growing patterns and shrinking patterns involving addition and subtraction, with and without the use of calculators * identify repeating, growing, and shrinking patterns found in real-life contexts * represent a given growing or shrinking   pattern in a variety of ways   * create growing or shrinking patterns * create a repeating pattern by combining   two attributes   * demonstrate, through investigation, an   understanding that a pattern results from  repeating an operation or making a repeated change to an attribute   * demonstrate an understanding of the concept   of equality by  partitioning whole  numbers to 18 in a  variety of ways, using  concrete materials;   * represent, through investigation with concrete materials and pictures, two number expressions that are equal, using the equal sign ; * determine the missing number in equations   involving addition and  subtraction to 18,  using a variety of tools and strategies   * identify, through investigation, and use the commutative property of addition to facilitate computation with whole numbers; * identify, through investigation, the properties of zero in addition and subtraction (i.e., when you add zero to a number, the number does not change; when you subtract zero from a number, the number does not change). * solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental strategies (Number Sense and Numeration) * describe relationships between quantities by using whole-number addition and subtraction (Number Sense and Numeration) * solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials (Number Sense and Numeration) * demonstrate an understanding of data displayed in a graph, by comparing different parts of the data and by making statements about the data as a whole (Data Management and Probability) | * identify, extend, and create a repeating   pattern involving two attributes, using a variety  of tools   * identify and describe, through investigation,   number patterns  involving addition,  subtraction, and  multiplication,  represented on a  number line, on a calendar, and on a hundreds;   * extend repeating, growing, and shrinking   number patterns   * create a number pattern involving addition or subtraction, given a pattern represented on a number line or a pattern rule expressed in words * represent simple geometric patterns using   a number sequence, a  number line, or a bar  graph;   * demonstrate, through investigation, an   understanding that a pattern results from  repeating an action , repeating an operation, using a transformation, or making some other repeated change to an  attribute   * determine, through investigation, the   inverse relationship between addition and subtraction ;   * determine, the missing number in equations   involving addition and subtraction of one- and two-digit numbers, using a variety of tools and strategies   * identify, through investigation, the properties of zero and one in multiplication (i.e.,   any number multiplied by zero equals zero; any number multiplied by 1 equals the original number)   * identify, through investigation, and use the associative property of addition to facilitate   computation with whole  numbers   * solve problems involving the addition and subtraction of two-digit numbers, using a variety of mental strategies; (Number Sense and Numeration) * add and subtract three-digit numbers, using concrete materials, student generated algorithms, and standard algorithms; (Number Sense and Numeration) * use estimation when solving problems involving addition and subtraction, to help judge the reasonableness of a solution; (Number Sense and Numeration) * interpret and draw conclusions from data presented in charts, tables, and graphs; (Data Management and Probability) |

**Appendix 2: Curriculum Connections Across the Grades: Junior**

Overview of the mathematical knowledge and skills that students are expected to acquire in the strand

***Patterning and Algebra: (Notice that some specific expectations related to Algebraic Reasoning but connected to other strands are included)***

**Grade 4:**

* relating the term and the term number in a numeric sequence;
* generating patterns that involve addition, subtraction, multiplication, and reflections;
* determining the missing numbers in equations involving multiplication of one- and two-digit numbers;
* using the commutative and distributive properties to facilitate computation

**Grade 5:**

* representing a pattern using a table of values;
* predicting terms in a pattern;
* determining the missing numbers in equations involving addition, subtraction, multiplication, or division and one- or two-digit numbers;
* investigating variables as unknown quantities;
* demonstrating equality using multiplication or division in equations with unknown quantities on both sides

**Grade 6:**

* representing patterns using ordered pairs and graphs;
* describing pattern rules in words;
* calculating any term when given the term number
* investigating variables as changing quantities;
* solving equations using concrete materials and guess and check

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| **Grade 4** | **Grade 5** | **Grade 6** |
| * extend, describe, and create repeating, growing, and shrinking number patterns; * connect each term in a growing or shrinking pattern with its term number, and record the patterns in a table of values that shows the term number and the term; * create a number pattern involving addition, subtraction, or multiplication, given a pattern rule expressed in words; * make predictions related to repeating geometric and numeric * extend and create repeating patterns that result from reflections, through investigation using a variety of tools. * determine, through investigation, the inverse relationship between multiplication and division; * determine the missing number in equations involving multiplication of one- and two-digit numbers, using a variety of tools and strategies * identify, through investigation, and use the distributive property of multiplication over addition to facilitate computation with whole * solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard   algorithms (Number Sense and Numeration)   * add and subtract decimal numbers to tenths, using concrete materials   and student-generated algorithms (Number Sense and Numeration) | * create, identify, and extend numeric and geometric patterns, using a variety of tools; * build a model to represent a number pattern presented in a table of values that shows the term number and the term; * make a table of values for a pattern that is generated by adding or subtracting a number   (i.e., a constant) to get the next  term, or by multiplying or dividing by a constant to get the next term, given either the sequence or the pattern rule in words;   * make predictions related to growing and shrinking geometric and numeric patterns * extend and create repeating patterns that result from translations, through investigation   using a variety of tools.   * demonstrate, through investigation, an understanding of variables as changing quantities, given equations with letters or   other symbols that describe  relationships involving simple ;   * demonstrate, through investigation, an   understanding of variables as  unknown quantities represented  by a letter or other symbol;   * determine the missing number in equations involving addition, subtraction, multiplication, or division and one- or two digit   numbers, using a variety of tools  and strategies   * solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies (Number Sense and Numeration) * add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms (Number Sense and Numeration) * describe multiplicative relationships between quantities by using simple fractions and decimals (Number Sense and Numeration) * determine, through investigation using a variety of tools and strategies the relationships between the length and width of a rectangle and its area and perimeter, and generalize to develop the formulas [i.e., *Area* = *length* x *width*; *Perimeter* = (2 x *length*) + (2 x w*idth*)];(Measurement) | * identify geometric patterns, through investigation using concrete materials or drawings, and represent them numerically; * make tables of values for growing patterns, given pattern rules in words, then list the ordered pairs (with the first coordinate representing the term number and the second coordinate representing the term) and plot the points in the first quadrant, using a   variety of tools;   * determine the term number of a given   term in a growing pattern that is  represented by a pattern rule in  words, a table of values, or a graph   * describe pattern rules (in words) that generate patterns by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term, then distinguish such pattern rules from pattern rules, given in words, that describe the general term by referring to the term number; * determine a term, given its term number, by extending growing and shrinking patterns that are generated by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term * extend and create repeating patterns that result from rotations, through investigation using a variety of tools. * demonstrate an understanding of different ways in which variables are; * identify, through investigation, the quantities in an equation that vary and those that remain constant; * solve problems that use two or three symbols or letters as variables to represent different unknown quantities * determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies * use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers; (Number Sense and Numeration) * solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools and strategies (Number Sense and Numeration) * add and subtract decimal numbers to   thousandths, using concrete  materials, estimation, algorithms, and  calculators; (Number Sense and  Numeration)   * multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators (Number Sense and Numeration) * use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution (Number Sense and Numeration) * develop the formulas for the area of a parallelogram (i.e., *Area of parallelogram* = *base* x *height*) and the area of a triangle [i.e., *Area of triangle* = (*base* x *height*) ÷ 2], using the area relationships among rectangles, parallelograms, and triangles (Measurement) |

**Appendix 2: Curriculum Connections Across the Grades: Intermediate**

Overview of the mathematical knowledge and skills that students are expected to acquire in the strand

***Patterning and Algebra: (Notice that some specific expectations related to Algebraic Reasoning but connected to other strands are included)***

**Grade 7:**

* representing linear growing patterns; representing patterns algebraically;
* modelling real-life relationships involving constant rates graphically and algebraically;
* translating phrases, using algebraic expressions; finding the term in a pattern algebraically when given any term number;
* solving linear equations using concrete materials or inspection and guess and check

**Grade 8:**

* representing the general term in a linear sequence, using one or more algebraic expressions;
* translating statements, using algebraic equations;
* finding the term number in a pattern algebraically when given any term;
* solving linear equations involving one variable terms with integer solutions using a “balance” model

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| **Grade 7** | **Grade 8** |
| * represent linear growing patterns, using a variety of   tools and strategies;   * make predictions about linear growing patterns, through investigation with concrete materials; * develop and represent the general term of a   linear growing pattern, using algebraic  expressions involving one operation;   * compare pattern rules that generate a pattern by adding or subtracting a constant, or multiplying or   dividing by a constant, to get the next term with pattern rules that use the term number to describe the general term;   * model everyday life relationships involving constant   rates, where the initial condition starts at 0, through investigation and using tables of values and graphs;   * model everyday life relationships involving constant   rates, using algebraic equations with variables to represent the changing quantities in the relationship;   * translate phrases describing simple mathematical   relationships into algebraic expressions, using concrete materials;   * evaluate algebraic expressions by substituting natural numbers for the variables; * make connections between evaluating algebraic expressions and determining the term in a pattern using the general term; * solve linear equations of the form *ax* = *c* or   *c* = *ax* and *ax* + *b* = *c* or variations such as  *b* + *ax* = *c* and *c* = *bx* + *a* (where *a*, *b*, and *c* are natural  numbers) by modelling with concrete materials, by  inspection, or by guess and check, with and  without the aid of a calculator.   * use a variety of mental strategies to solve problems involving the addition and subtraction of fractions and decimals (Number Sense and Numeration) * solve problems involving the multiplication and division of decimal numbers to thousandths by one-digit whole numbers, using a variety of tools and strategies (Number Sense and Numeration) * use estimation when solving problems involving operations with whole numbers, decimals, and percents, to help judge the reasonableness of a solution (Number Sense and Numeration) * add and subtract fractions with simple like and unlike denominators, using a variety of tools and algorithms; (Number Sense and Numeration) * determine, through investigation using a variety of tools   and strategies the relationship for calculating the area of a trapezoid, and generalize to develop the formula [i.e., *Area* = (*sum of lengths of parallel sides* x *height*) ÷ 2] (Measurement) | * represent, through investigation with concrete materials, the general term of a linear pattern, using   one or more algebraic expressions;   * represent linear patterns graphically, using a variety of tools; * determine a term, given its term number, in a linear pattern that is represented by a graph or an algebraic   equation;   * describe different ways in which algebra can be used in everyday life situations; * model linear relationships using tables of values,   graphs, and equations, through investigation and  using a variety of tools;   * translate statements describing mathematical   relationships into algebraic expressions and  equations;   * evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables; * make connections between solving equations and   determining the term number in a pattern, using the  general term;   * solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a “balance” model * determine common factors and common multiples using the prime factorization of numbers; (Number Sense and Numeration) * solve multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools and strategies (Number Sense and Numeration) * use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions, to help judge the reasonableness of a solution; (Number Sense and Numeration) * evaluate expressions that involve integers, including expressions that contain brackets and exponents, using order of operations; (Number Sense and Numeration) * determine, through investigation using a variety of tools and strategies, the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas [i.e., *Circumference of a circle* = π x *diameter*; *Area of a circle* = π x (*radius*)²] (Measurement) * determine, through investigation using a variety of tools and strategies, the relationship between the area of the base and height and the volume of a cylinder, and generalize to develop the formula (i.e.,*Volume* = *area of base* x *height*); (Measurement) * determine the Pythagorean relationship, through investigation using a variety of tools and strategies; (Geometry and Spatial Sense) * determine, through investigation using concrete materials, the relationship between the numbers of faces, edges, and vertices of a polyhedron (i.e., *number of faces + number of vertices* = *number of edges +* 2) (Geometry and Spatial Sense) |

**Appendix 2: Curriculum Connections Across the Grades: Intermediate**

Overview of the mathematical knowledge and skills related to Algebraic Reasoning that students are expected to acquire in the strands

***Number Sense and Algebra, Linear Relations, and Measurement and Geometry:***

**Grade 9 Applied:**

* simplify numerical and polynomial expressions in one variable, and solve simple first-degree

equations.

* determine the characteristics of linear relations;
* demonstrate an understanding of constant rate of change and its connection to linear

relations;

* connect various representations of a linear relation, and solve problems using the

representations.

* solve problems involving the measurements of two-dimensional shapes and the volumes of

three-dimensional figures

Overview of the mathematical knowledge and skills related to Algebraic Reasoning that students are expected to acquire in the strands

***Number Sense and Algebra, Linear Relations, Analytic Geometry, and Measurement and Geometry:***

**Grade 9 Academic:**

* demonstrate an understanding of the exponent rules of multiplication and division, and

apply them to simplify expressions;

* manipulate numerical and polynomial expressions, and solve first-degree equations
* demonstrate an understanding of the characteristics of a linear relation;
* connect various representations of a linear relation.
* determine the relationship between the form of an equation and the shape of its graph with

respect to linearity and non-linearity;

* solve problems involving the measurements of two-dimensional shapes and the surface areas

and volumes of three-dimensional figures

Overview of the mathematical knowledge and skills related to Algebraic Reasoning that students are expected to acquire in the strands

***Measurement and Geometry, Modelling Linear Relations, and* Quadratic Relations of the Form *y* = *ax*² + *bx* + *c***

**Grade 10 Applied:**

* solve problems involving right triangles, using the primary trigonometric ratios and the

Pythagorean theorem;

* solve problems involving the surface areas and volumes of three-dimensional figures, and

use the imperial and metric systems of measurement.

* manipulate and solve algebraic equations, as needed to solve problems;
* graph a line and write the equation of a line from given information;
* solve systems of two linear equations, and solve related problems that arise from realistic

situations.

* manipulate algebraic expressions, as needed to understand quadratic relations;
* identify characteristics of quadratic relations;

Overview of the mathematical knowledge and skills related to Algebraic Reasoning that students are expected to acquire in the strands

**Quadratic Relations of the Form *y* = *ax*² + *bx* + *c, Analytic Geometry, and Trigonometry***

**Grade 10 Academic:**

* relate transformations of the graph of *y* = *x*2 to the algebraic representation

*y* = *a*(*x – h*)2 *+ k*;

* solve quadratic equations and interpret the solutions with respect to the corresponding

relations;

* solve problems involving quadratic relations
* model and solve problems involving the intersection of two straight lines;
* solve problems using analytic geometry involving properties of lines and line segments;
* solve problems involving right triangles, using the primary trigonometric ratios and the

Pythagorean Theorem;

* solve problems involving acute triangles, using the sine law and the cosine law.

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| **Grade 9** | **Grade 10** |
| **Applied**   * solve for the unknown value in a proportion, using a variety of methods; * multiply a polynomial by a monomial involving the same variable to give results up to degree three, using a variety of tools; * carry out an investigation or experiment involving   relationships between two variables, including the  collection and organization of data, using appropriate  methods, equipment, and/or technology and  techniques.   * solve problems involving ratios, rates, and directly proportional relationships in various contexts using a variety of methods * simplify numerical expressions involving integers and rational numbers, with and without the use of technology * relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve   equations;   * describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three * substitute into and evaluate algebraic expressions   involving exponents (i.e., evaluate expressions  involving natural-number exponents with rational-  number bases)   * add and subtract polynomials involving the same variable up to degree three, using a variety of tools * multiply a polynomial by a monomial involving the same variable to give results up to degree three, using a variety of tools * solve first-degree equations with non-fractional   coefficients, using a variety of tools and strategies   * substitute into algebraic equations and solve for one variable in the first degree * pose problems, identify variables, and formulate   hypotheses associated with relationships between  two variables   * compare the properties of direct variation and partial variation in applications, and identify the initial value * express a linear relation as an equation in two variables, using the rate of change and the initial value * determine other representations of a linear relation arising from a realistic situation, given one representation * solve problems that can be modelled with first-degree equations, and compare the algebraic method to other solution methods * relate the geometric representation of the Pythagorean theorem to the algebraic representation *a*² + *b*² = *c*²   **Academic**   * substitute into and evaluate algebraic expressions   involving exponents (i.e., evaluate expressions  involving natural-number exponents with rational-  number bases   * describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by *x*; area, which is two dimensional, can be represented by (*x*)(*x*) or *x*²; volume, which is three dimensional, can be represented by (*x*)(*x*)(*x*), (*x*2)(*x*), or *x*³]; * derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents; * extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents; * simplify numerical expressions involving integers and rational numbers, with and without the use of technology; relate their understanding of inverse * operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations; * add and subtract polynomials with up to two variables, using a variety of tools * multiply a polynomial by a monomial involving the same variable, using a variety of tools; * expand and simplify polynomial expressions involving one variable, using a variety of tools; * solve first-degree equations, including equations with fractional coefficients, using a variety of tools and   strategies;   * rearrange formulas involving variables in the first degree, with and without substitution; * solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods. * use patterning to derive the multiplication, division, and power exponent laws; * multiply a polynomial by a monomial involving the same variable; * determine other representations of a linear relation, given one representation; * design and carry out an investigation or experiment   involving relationships between two variables,  including the collection and organization of data, using appropriate methods, equipment, and/or technology and techniques;   * identify *y* = *mx* + *b* as a common form for the equation of a straight line; * identify properties of the slopes of lines and line   segments;   * express the equation of a line in the form *y* = *mx* + *b*, given the form *Ax* + *By* + *C* = 0. * relate the geometric representation of the Pythagorean theorem and the algebraic representation *a*² + *b*² = *c*²; | **Applied**   * verify, through investigation, properties of similar triangles solve problems involving similar triangles in realistic situations * solve problems involving the surface areas of prisms, pyramids, and cylinders, and volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving combinations of these figures, using the metric system or the imperial * solve first-degree equations involving one variable, including equations with fractional coefficients * determine the value of a variable in the first degree, using a formula (i.e., by isolating the variable and then substituting known values; by substituting known values and then solving for the variable) e * express the equation of a line in the form *y* = *mx* + *b*, given the form *Ax* + *By* + *C* = 0. * identify, through investigation, *y* = *mx* + *b* as a common form for the equation of a straight line, and identify the special cases *x* = *a*, *y* = *b*; * determine the equation of a line, given its graph, the slope and *y*-intercept, the slope and a point on the line, or two points on the line. * solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination * solve problems that arise from realistic situations described in words or represented by given linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method * expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial using a variety of tools and strategies; * factor binomials and trinomials involving one variable up to degree two, by determining a common factor using a variety of tools and strategies; * factor simple trinomials of the form *x*² + *bx* + *c* , using a variety of tools and strategies * factor the difference of squares of the form *x*² *– a*² * compare, through investigation using technology, the graphical representations of a quadratic relation in the form *y* = *x*² + *bx* + *c* and the same relation in the factored form *y* = (*x – r*)(*x – s*) (i.e., the graphs are the same), and describe the connections between each algebraic representation and the graph   **Academic**   * compare, through investigation using technology, the features of the graph of *y* = *x*² and the graph of *y* = 2*x*, and determine the meaning of a negative exponent and of zero as an exponent * identify, through investigation using technology, the effect on the graph of *y* = *x* of transformations (i.e., translations, reflections in the *x*-axis, vertical stretches or compressions) by considering separately each parameter *a*, *h*, and *k* [i.e., investigate the effect on the graph of *y* = *x*² of *a*, *h*, and *k* in *y* = *x*² *+ k*, *y* = (*x* – *h*)², and *y* = *ax*²]; * explain the roles of *a*, *h*, and *k* in *y* = *a*(*x* – *h* )² + *k*, using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry; * determine the equation, in the form *y* = *a*(*x* – *h*)² + *k*, of a given graph of a parabola. * expand and simplify second-degree polynomial expressions, using a variety of tools and strategies; * factor polynomial expressions involving common factors, trinomials, and differences of squares, using a variety of tools and strategies; * determine, through investigation, and describe the connection between the factors of a quadratic expression and the *x*-intercepts (i.e., the zeros) of the graph of the corresponding quadratic relation, expressed in the form *y* = *a*(*x* – *r*)(*x* – *s*); * interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the *x*-intercepts of the corresponding relations; * express *y* = *ax*² + *bx* + *c* in the form *y* = *a*(*x* – *h*)² + *k* by completing the square in situations involving no fractions, using a variety of tools; * explore the algebraic development of the quadratic formula * solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing) * solve systems of two linear equations involving two variables, using the algebraic method of substitution or elimination * solve problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method * develop the formula for the midpoint of a line segment, and use this formula to solve problems * develop the formula for the length of a line segment, and use this formula to solve problems * develop the equation for a circle with centre (0, 0) and radius *r*, by applying the formula for the length of a line segment; * verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures * plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property * solve problems involving right triangles, using the primary trigonometric ratios and the Pythagorean theorem; * solve problems involving acute triangles, using the sine law and the cosine law. * verify, through investigation, the properties of similar triangles; * describe and compare the concepts of similarity and congruence; * solve problems involving similar triangles in realistic situations; * determine, through investigation, the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios |

**Appendix 2: Curriculum Connections Across the Grades: Senior**

The mathematical knowledge and skills related to Algebraic Reasoning that students are expected to acquire are found across the strands

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| **Grade 11** | |
| **MCR3U Functions (University)**  **Characteristics of Functions:**   * explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies * represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of the functions *f*(*x*) = *x*, *f*(*x*) = *x* , *f*(*x*) = √*x,* and *f*(*x*) = ; describe the domain and range of the function appropriately * relate the process of determining the inverse of a function to their understanding of reverse processes * determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical, or algebraic representation of the function, and make connections, through investigation using a variety of tools, between the graph of a function and the graph of its inverse * determine, through investigation, the relationship between the domain and range of a function and the domain and range of the inverse relation, and determine whether or not the inverse relation is a function * determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function, and make connections, through investigation using a variety of tools between the algebraic representations of a function and its inverse * sketch graphs of *y* = *af* (*k*(*x* – *d*)) + *c* by applying one or more transformations to the graphs of *f*(*x*) = *x*, *f*(*x*) = *x²* , *f*(*x*) = √*x*,, and state the domain and range of the transformed functions determine the number of zeros (i.e., *x*-intercepts) of a quadratic function, using a variety of strategies * determine the maximum or minimum value of a quadratic function whose equation is given in the form *f*(*x*) = *ax²*+ *bx* + *c*, using an algebraic method * determine, through investigation, the transformational relationship among the family of quadratic functions that have the same zeros, and determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function * solve problems involving the intersection of a linear function and a quadratic function graphically and algebraically * simplify polynomial expressions by adding, subtracting, and multiplying * determine if two given algebraic expressions are equivalent   **Exponential Functions:**   * simplify algebraic expressions containing integer and rational exponents determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, for exponential functions represented in a variety of ways * distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways * sketch graphs of *y* = *af* (*k*(*x* – *d*)) + *c* by applying one or more transformations to the graph of *f*(*x*) = *a* (*a* > 0, *a* ≠ 1), and state the domain and range of the transformed functions * determine, through investigation using technology, that the equation of a given exponential function can be expressed using different bases and explain the connections between the equivalent forms in a variety of ways * represent an exponential function with an equation, given its graph or its properties * identify exponential functions, including those that arise from real-world applications involving growth and decay, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range * solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications   **Trigonometric Functions:**   * prove simple trigonometric identities, using the Pythagorean identity sin *x²* + cos *x²* = 1; the quotient identity  ; and the reciprocal identities * pose problems involving right triangles and oblique triangles in two dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case) * pose problems involving right triangles and oblique triangles in three-dimensional settings, and solve these and other such problems using the primary trigonometric ratios * determine, through investigation using technology, the roles of the parameters *a*, *k*, *d*, and *c* in functions of the form *y* =*af* (*k*(*x – d*)) + *c*, where *f*(*x*) =sin*x* or *f*(*x*) =cos*x* with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of *f*(*x*) =sin*x* and *f*(*x*) =cos*x* (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the *x*- and *y*-axes) * represent a sinusoidal function with an equation, given its graph or its properties * determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles * predict the effects on a mathematical model (i.e., graph, equation) of an application involving periodic phenomena when the conditions in the application are varied * pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation   **Discrete Functions:**   * connect the formula for the *n*th term of a sequence to the representation in function notation, and write terms of a sequence given one of these representations or a recursion formula * represent a sequence algebraically using a recursion formula, function notation, or the formula for the *n*th term and describe the information that can be obtained by inspecting each representation * determine, through investigation, recursive patterns in the Fibonacci sequence, in related sequences, and in Pascal’s triangle, and represent the patterns in a variety of ways * determine, through investigation, and describe the relationship between Pascal’s triangle and the expansion of binomials, and apply the relationship to expand binomials raised to whole-number exponents * identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representation * determine the formula for the general term of an arithmetic sequence [i.e., *tn* = *a* + (*n* –1)*d* ] or geometric sequence (i.e., *tn* = *ar n* – 1), through investigation using a variety of tools and strategies,   and apply the formula to calculate any term in a  sequence   * determine the formula for the sum of an arithmetic or geometric series, through investigation using a variety of tools and strategies, and apply the formula to calculate the sum of a given number of consecutive terms | **MCF3M Functions and Applications (University/College)**  **Quadratic Functions:**   * represent situations using quadratic expressions in one variable, and expand and simplify quadratic expressions in one variable * factor quadratic expressions in one variable, including those for which *a* ≠ 1 * solve quadratic equations by selecting and applying a factoring strategy * determine, through investigation, and describe the connection between the factors used in solving a quadratic equation and the *x*-intercepts of the graph of the corresponding quadratic equations * explore the algebraic development of the quadratic formula, and apply the formula to solve quadratic equations, using technology * relate the real roots of a quadratic equation to the *x*-intercepts of the corresponding graph, and connect the number of real roots to the value of the discriminant * determine the real roots of a variety of quadratic equations, and describe the advantages and disadvantages of each strategy (i.e., graphing; factoring; using the quadratic formula) * explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies * substitute into and evaluate linear and quadratic functions represented using function notation, including functions arising from real-world applications * explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately * express the equation of a quadratic function in the standard form *f* (*x*) = *ax²* + *bx* + *c*, given the vertex form *f* (*x*) = *a*(*x – h*)² + *k*, and verify, using graphing technology, that these forms are equivalent representations * express the equation of a quadratic function in the vertex form *f*(*x*) = *a*(*x – h*)² + *k*, given the standard form *f*(*x*) = *ax²* + *bx* + *c*, by completing the square including cases where is a simple rational number, and verify, using graphing technology, that these forms are equivalent representations * sketch graphs of quadratic functions in the factored form *f*(*x*) = *a*(*x – r* )(*x – s*) by using the *x*-intercepts to determine the vertex * describe the information that can be obtained by inspecting the standard form *f*(*x*) = *ax²* + *bx* + *c*, the vertex form *f*(*x*) = *a*(*x – h*)² + *k*, and the factored form *f*(*x*) = *a*(*x – r*)(*x – s*) of a quadratic function * sketch the graph of a quadratic function whose equation is given in the standard form *f*(*x*) = *ax²* + *bx* + *c* by using a suitable strategy, and identify the key features of the graph * determine, through investigation using a variety of strategies , the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology * solve problems arising from real-world applications, given the algebraic representation of a quadratic function   **Exponential Functions:**   * evaluate, with and without technology, numerical expressions containing integer and rational exponents and rational bases * graph, with and without technology, an exponential relation, given its equation in the form (*a* > 0, *a* ≠ 1), define this relation as the function , and explain why it is a function * determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals and asymptotes for exponential functions represented in a variety of ways * distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways, within the same context when possible * identify exponential functions, including those that arise from real-world applications involving growth and decay, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range * solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications by interpreting the graphs or by substituting values for the exponent into the equations   **Trigonometric Functions:**   * solve problems that require the use of the sine law or the cosine law in acute triangles, including problems arising from real-world applications * make connections between the sine ratio and the sine function by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios, with or without technology, defining this relationship as the function *f*(*x*) = sin*x*, and explaining why the relationship is a function * determine, through investigation using technology, the roles of the parameters *a*, *c*, and *d* in functions in the form *f*(*x*) = *a* sin*x*, *f*(*x*) = sin*x* + *c*, and *f*(*x*) = sin(*x – d*), and describe these roles in terms of transformations on the graph of *f*(*x*) = sin*x* with angles expressed in degrees (i.e., translations; reflections in the *x*-axis; vertical stretches and compressions to and from the *x*-axis) * sketch graphs of *f*(*x*) = *a* sin*x*, *f*(*x*) = sin*x* + *c*, and *f*(*x*) = sin(*x – d*) by applying transformations to the graph of *f*(*x*) = sin*x*, and state the domain and range of the transformed functions * identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range * pose problems based on applications involving a sine function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation |
| **MBF3C Foundations for College Mathematics (College)**  **Mathematical Models:**   * sketch graphs of quadratic relations represented by the equation *y* = *a*(*x* – *h*)² + *k* * expand and simplify quadratic expressions in one variable involving multiplying binomials express the equation of a quadratic relation in the standard form *y* = *ax²* + *bx* + *c*, given the vertex form *y* = *a*(*x* – *h*)² + *k*, and verify, using graphing technology, that these forms are equivalent representations * factor trinomials of the form *ax²* + *bx* + *c*, where *a* = 1 or where *a* is the common factor, by various methods * determine, through investigation, and describe the connection between the factors of a quadratic expression and the *x*-intercepts of the graph of the corresponding quadratic relation * solve problems, using an appropriate strategy (i.e., factoring, graphing), given equations of quadratic relations, including those that arise from real-world applications * graph simple exponential relations, using paper and pencil, given their equations make and describe connections between representations of an exponential relation * distinguish exponential relations from linear and quadratic relations by making comparisons in a variety of ways, within the same context when possible * pose problems involving exponential relations arising from a variety of real-world applications, and solve these and other such problems by using a given graph or a graph generated with technology from a given table of values or a given equation * pose problems involving exponential relations arising from a variety of real-world applications, and solve these and other such problems by using a given graph or a graph generated with technology from a given table of values or a given equation   **Personal Finance:**   * calculate the total interest earned on an investment or paid on a loan by determining the difference between the amount and the principal * solve problems involving applications of the compound interest formula to determine the cost of making a purchase on credit * solve problems, using technology, that involve the fixed costs and variable costs of owning and operating a vehicle   **Geometry and Trigonometry:**   * solve problems, including those that arise from real-world applications, by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios * solve problems that arise from real-world applications involving metric and imperial measurements and that require the use of the sine law or the cosine law in acute triangles | **MEL3E Mathematics for Work and Everyday Life (Workplace)** |

**Appendix 2: Curriculum Connections Across the Grades: Senior**

The mathematical knowledge and skills related to Algebraic Reasoning that students are expected to acquire are found across the strands

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| **Grade 12** | | |
| **MHF4U Advanced Functions**  **(University)**  **Exponential and Logarithmic Functions:**   * make connections between related logarithmic and exponential equations and solve simple exponential equations by rewriting them in logarithmic form * determine, through investigation with technology and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, increasing/decreasing behaviour) of the graphs of logarithmic functions of the form , and make connections between the algebraic and graphical representations of these logarithmic functions * recognize the relationship between an exponential function and the corresponding logarithmic function to be that of a function and its inverse, deduce that the graph of a logarithmic function is the reflection of the graph of the corresponding exponential function in the line *y* = *x*, and verify the deduction using technology * pose problems based on real-world applications of exponential and logarithmic functions, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation * recognize equivalent algebraic expressions involving logarithms and exponents, and simplify expressions of these types * solve exponential equations in one variable by determining a common base and by using logarithms (sides), recognizing that logarithms base 10 are commonly used * solve simple logarithmic equations in one variable algebraically * solve problems involving exponential and logarithmic equations algebraically, including problems arising from real-world applications   **Trigonometric Functions:**   * represent a sinusoidal function with an equation, given its graph or its properties, with angles expressed in radians * pose problems based on applications involving a trigonometric function with domain expressed in radians, and solve these and other such problems by using a given graph or a graph generated with or without technology from a table of values or from its equation * recognize equivalent trigonometric expressions equivalent, and verify equivalence using graphing technology * explore the algebraic development of the compound angle formulas and use the formulas to determine exact values of trigonometric ratios, * recognize that trigonometric identities are equations that are true for every value in the domain (i.e., a counter-example can be used to show that an equation is not an identity), * prove trigonometric identities through the application of reasoning skills, using a variety of relationships, and verify identities using technology * solve linear and quadratic trigonometric equations, with and without graphing technology, for the domain of real values from 0 to 2π, and solve related problems   **Polynomials and Rational Functions:**   * recognize a polynomial expression (i.e., a series of terms where each term is the product of a constant and a power of *x* with a nonnegative integral exponent, such as *x³* – 5*x²* + 2*x* – 1); * recognize the equation of a polynomial function, give reasons why it is a function, and identify linear and quadratic functions as examples of polynomial functions compare, through investigation using graphing technology, the numeric, graphical, and algebraic representations of polynomial (i.e., linear, quadratic, cubic, quartic) functions * distinguish polynomial functions from sinusoidal and exponential functions, and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions * determine an equation of a polynomial function that satisfies a given set of conditions, using methods appropriate to the situation, and recognize that there may be more than one polynomial function that can satisfy a given set of conditions; * determine the equation of the family of polynomial functions with a given set of zeros and of the member of the family that passes through another given point determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions * determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that have linear expressions in the numerator and denominator, and make connections between the algebraic and graphical representations of these rational functions * sketch the graph of a simple rational function using its key features, given the algebraic representation of the function * factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring, factoring by grouping, remainder theorem, factor theorem) * determine, through investigation using technology, the connection between the real roots of a polynomial equation and the *x*-intercepts of the graph of the corresponding polynomial function, and describe this connection * solve polynomial equations in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring, factoring by grouping, remainder theorem, factor theorem), and verify solutions using technology determine, through investigation using technology, the connection between the real roots of a rational equation and the *x*-intercepts of the graph of the corresponding rational function, and describe this connection * solve simple rational equations in one variable algebraically, and verify solutions using technology solve problems involving applications of polynomial and simple rational functions and equations * explain, for polynomial and simple rational functions, the difference between the solution to an equation in one variable and the solution to an inequality in one variable, and demonstrate that given solutions satisfy an inequality * determine solutions to polynomial inequalities in one variable and to simple rational inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which *x* satisfies the inequalities solve linear inequalities and factorable polynomial inequalities in one variable in a variety of ways, and represent the solutions on a number line or algebraically * explain, for polynomial and simple rational functions, the difference between the solution to an equation in one variable and the solution to an inequality in one variable, and demonstrate that given solutions satisfy an inequality; * determine solutions to polynomial inequalities in one variable and to simple rational inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which *x* satisfies the inequalities * solve linear inequalities and factorable polynomial inequalities in one variable in a variety of ways, and represent the solutions on a number line or algebraically   **Characteristics of Functions:**   * gather, interpret, and describe information about real-world applications of rates of change, and recognize different ways of representing rates of change * recognize that the rate of change for a function is a comparison of changes in the dependent variable to changes in the independent variable, and distinguish situations in which the rate of change is zero, constant, or changing by examining applications, including those arising from real-world situations * calculate and interpret average rates of change of functions arising from real-world applications, given various representations of the functions * determine, through investigation using various representations of relationships, approximate instantaneous rates of change arising from real-world applications by using average rates of change and reducing the interval over which the average rate of change is determined * make connections, through investigation, between the slope of a secant on the graph of a function and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph of a function and the instantaneous rate of change of the function at that point * determine, through investigation using graphing technology, key features of the graphs of functions created by adding, subtracting, multiplying, or dividing functions and describe factors that affect these properties * recognize real-world applications of combinations of functions, and solve related problems graphically * determine, through investigation, and explain some properties (i.e., odd, even, or neither; increasing/decreasing behaviours) of functions formed by adding, subtracting, multiplying, and dividing general functions determine the composition of two functions [i.e., *f*(*g*(*x*))] numerically (i.e., by using a table of values) and graphically, with technology, for functions represented in a variety of ways, and interpret the composition of two functions in real-world applications * determine algebraically the composition of two functions [i.e., *f*(*g*(*x*))], verify that *f*(*g*(*x*)) is not always equal to *g*( *f*(*x*)), and state the domain [i.e., by defining *f*(*g*(*x*)) for those *x*-values for which *g*(*x*) is defined and for which it is included in the domain of *f*(*x*)] and the range of the composition of two functions * solve problems involving the composition of two functions, including problems arising from real-world applications * demonstrate, by giving examples for functions represented in a variety of ways, the property that the composition of a function and its inverse function maps a number onto itself [i.e., *f* ( *f*(*x*)) = *x* and *f*( *f* (*x*)) = *x* * demonstrate that the inverse function is the reverse process of the original function and that it undoes what the function does * compare, through investigation using a variety of tools and strategies the characteristics of various functions | **MCV4U Calculus and Vectors**  **(University)**  **Rate of Change:**   * describe examples of real-world applications of rates of change, represented in a variety of ways * describe connections between the average rate of change of a function that is smooth (i.e., continuous with no corners) over an interval and the slope of the corresponding secant, and between the instantaneous rate of change of a smooth function at a point and the slope of the tangent at that point * make connections, for a function that is smooth over the interval *a* ≤ *x* ≤ *a* + *h*, between the average rate of change of the function over this interval and the value of the expression *,* and between the instantaneous rate of change of the function at *x* = *a* and the value of the * determine numerically and graphically the intervals over which the instantaneous rate of change is positive, negative, or zero for a function that is smooth over these intervals, and describe the behaviour of the instantaneous rate of change at and between local maxima and minima * generate, through investigation using technology, a table of values showing the instantaneous rate of change of a polynomial function, *f*(*x*), for various values of *x*, graph the ordered pairs, recognize that the graph represents a function called the derivative, *f* ’(*x*) or , and make connections between the graphs of *f*(*x*) and *f* ’(*x*) or *y* and * determine the derivatives of polynomial functions by simplifying the algebraic expression and then taking the limit of the simplified expression as *h* approaches zero [i.e., determining ] * determine, through investigation using technology the graph of the derivative *f* ’(*x*) or of a given sinusoidal function [i.e., *f*(*x*) = sin *x*, *f*(*x*) = cos *x*] * determine, through investigation using technology, the graph of the derivative *f* ’(*x*) or of a given exponential function [i.e., *f*(*x*) = *a* (*a* > 0, *a* ≠ 1)] and make connections between the graphs of *f*(*x*) and *f* ’(*x*) or *y* and * determine, through investigation using technology, the exponential function (*a* > 0, *a* ≠ 1) for which *f* ’(*x*) = *f*(*x*), identify the number *e* to be the value of *a* for which *f* ’(*x*) = *f*(*x*) [i.e., given , ], and recognize that for the exponential function the slope of the tangent at any point on the function is equal to the value of the function at that point * verify the power rule for functions of the form , where *n* is a natural number * determine algebraically the derivatives of polynomial functions, and use these derivatives to determine the instantaneous rate of change at a point and to determine point(s) at which a given rate of change occurs * verify that the power rule applies to functions of the form *f*(*x*) = *x* , where *n* is a rational number, and verify algebraically the chain rule using monomial functions and the product rule using polynomial functions * solve problems, using the product and chain rules, involving the derivatives of polynomial functions, sinusoidal functions, exponential functions, rational functions   Derivatives and Their Applications:   * determine algebraically the equation of the second derivative *f* ”(*x*) of a polynomial or simple rational function *f*(*x*), and make connections, through investigation using technology, between the key features of the graph of the function and corresponding features of the graphs of its first and second derivatives * describe key features of a polynomial function, given information about its first and/or second derivatives, sketch two or more possible graphs of the function that are consistent with the given information, and explain why an infinite number of graphs is possible * sketch the graph of a polynomial function, given its equation, by using a variety of strategies to determine its key features, and verify using technology   **Geometry and Algebra of Vectors:**   * determine, using trigonometric relationships, the Cartesian representation of a vector in two-space given as a directed line segment, or the representation as a directed line segment of a vector in two-space given in Cartesian   form.   * recognize that points and vectors in three-space can both be represented using Cartesian coordinates, and determine the distance between two points and the magnitude of a vector   using their Cartesian representations   * perform the operations of addition, subtraction, and scalar multiplication on vectors represented as directed line segments in two space, and on vectors represented in Cartesian form in two-space and three-space * solve problems involving the addition, subtraction, and scalar multiplication of vectors, including problems arising from real-world applications * perform the operation of dot product on two vectors represented as directed line segments (i.e., using *a*•*b* =|*a*||*b*|cosƟ) and in Cartesian form (i.e., using  *or*  in two-space and three-space, and describe applications of the dot product * determine, through investigation, properties of the dot product * perform the operation of cross product on two vectors represented in Cartesian form in three-space [i.e., using], * determine the magnitude of the cross product (i.e., using ), and describe   applications of the cross product   * determine, through investigation, properties of the cross product * determine, through investigation with technology (i.e., 3-D graphing software) and without technology, that the solution points (*x*, *y*, *z*) in three-space of a single linear equation in three variables form a plane and that the solution points (*x*, *y*, *z*) in three-space of a system of two linear equations in three variables form the line of intersection of two planes, if the   planes are not coincident or parallel   * recognize a scalar equation for a line in two-space to be an equation of the form *Ax* + *By* + *C* = 0, * represent a line in two-space using a vector equation (i.e., ) and parametric equations, and make connections between a scalar equation, a vector equation, and parametric equations of a line in two-space * recognize that a line in three-space cannot be represented by a scalar equation, and represent a line in three-space using the scalar equations of two intersecting planes and using vector and parametric equations * recognize a normal to a plane geometrically (i.e., as a vector perpendicular to the plane) and algebraically and determine, through investigation, some geometric properties of the plane * recognize a scalar equation for a plane in three-space to be an equation of the form *Ax* + *By* + *Cz* + *D* = 0 whose solution points make up the plane, determine the intersection of three planes represented using scalar equations by solving a system of three linear equations in three unknowns algebraically, and make connections between the algebraic solution and the geometric configuration of the three planes * determine, using properties of a plane, the scalar, vector, and parametric equations of a plane | **MDM4U Mathematics of Data**  **Management (University)**  **Counting and Probability:**   * determine the theoretical probability, *P* (i.e., a value from 0 to 1), of each outcome of a discrete sample space, * recognize that the sum of the probabilities of the outcomes is 1 (i.e., for *n* outcomes, ),recognize that the probabilities form the probability distribution associated with the sample space, and solve related problems * recognize and describe an event as a set of outcomes and as a subset of a sample space, determine the complement of an event, determine whether two or more events are mutually exclusive or non-mutually exclusive (and solve related probability problems using a variety of strategies * determine whether two events are independent or dependent and whether one event is conditional on another event, and solve related probability problems using avariety of strategies * recognize the use of permutations and combinations as counting techniques with advantages over other counting techniques, distinguish between situations that involve the use of permutations and those that involve the use of combinations, and make connections between, and calculate, permutations and combinations * solve simple problems using techniques for counting permutations and combinations, where all objects are distinct, and express the solutions using standard combinatorial notation * solve introductory counting problems involving the additive counting principle and the multiplicative counting principle * solve probability problems using counting principles for situations involving equally likely outcomes   **Probability Distributions:**   * recognize and identify a discrete random variable *X* (i.e., a variable that assumes a unique value for each outcome of a discrete sample space, such as the value *x* for the outcome of getting *x* heads in 10 tosses of a coin), generate a probability distribution [i.e., a function that maps each value *x* of a random variable *X* to a corresponding probability, *P*(*X*= *x*)] by calculating the probabilities associated with all values of a random variable, with and without technology, and represent a probability distribution numerically using a table * calculate the expected value for a given probability distribution [i.e., using *E*(*X*)= Σ *xP*(*X*= *x*)], interpret the expected value in applications, and make connections between the expected value and the weighted mean of the values of the discrete random variable * recognize conditions that give rise to a random variable that follows a binomial probability distribution, calculate the probability associated with each value of the random variable, represent the distribution numerically using a table and graphically using a probability histogram, and make connections to the algebraic representation *P*(*X*= *x*)=( )*p* (1 – *p*) * recognize and identify a continuous random variable (i.e., a variable that assumes values from the infinite number of possible outcomes in a continuous sample space), and distinguish between situations that give rise to discrete frequency distributions and situations that give rise to continuous frequency distributions * recognize standard deviation as a measure of the spread of a distribution, and determine, with and without technology, the mean and standard deviation of a sample of values of a continuous random variable * recognize a *z*-score as the positive or negative number of standard deviations from the mean to a value of the continuous random variable, and solve probability problems involving normal distributions using a variety of tools and strategies, including problems arising from real-world applications * recognize that the analysis of one-variable data involves the frequencies associated with one attribute, and determine, using technology, the relevant numerical summaries (i.e., mean, median, mode, range, interquartile range, variance, and standard deviation) * interpret statistical summaries to describe the characteristics of a two variable data set and to compare two related two-variable data sets; describe how statistical summaries can be used to misrepresent two-variable data; and make inferences, and make and justify conclusions, from statistical summaries of two-variable data orally and in writing, using convincing arguments   **Statistical Analysis:**   * recognize that the analysis of two-variable data involves the relationship between two attributes, recognize the correlation coefficient as a measure of the fit of the data to a linear model, and determine, using technology, the relevant numerical summaries * recognize and distinguish different types of relationships between two variables that have a mathematical correlation * determine, by performing a linear regression using technology, the equation of a line that models a suitable two-variable data set, determine the fit of an individual data point to the linear model, and recognize these processes as strategies for two-variable data analysis * interpret statistical summaries to describe the characteristics of a two-variable data set and to compare two related two-variable data sets; describe how statistical summaries can be used to misrepresent two-variable data; and make inferences, and make and justify conclusions, from statistical summaries of two-variable data orally and in writing, using convincing arguments |
| **MCT4C Mathematics for College Technology (College)**  **Exponential Functions:**   * solve simple exponential equations numerically and graphically, with technology, and recognize that the solutions may not be exact * determine, through investigation using graphing technology, the point of intersection of the graphs of two exponential functions, recognize the *x*-coordinate of this point to be the solution to the corresponding exponential equation, and solve exponential equations graphically or from its equation * simplify algebraic expressions containing integer and rational exponents using the laws of exponents * solve exponential equations in one variable by determining a common base * recognize the logarithm of a number to a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation (i.e., the undoing or reversing) of exponentiation, and evaluate simple logarithmic expressions * make connections between related logarithmic and exponential equations, and solve simple exponential equations by rewriting them in logarithmic form * pose problems based on real-world applications that can be modelled with given exponential equations, and solve these and other such problems algebraically by rewriting them in logarithmic form   **Polynomial Functions:**   * recognize a polynomial expression (i.e., a series of terms where each term is the product of a constant and a power of *x* with a nonnegative integral exponent, such as *x³* – 5*x²* + 2*x* – 1); * recognize the equation of a polynomial function and give reasons why it is a function, and identify linear and quadratic functions as examples of polynomial functions * compare, through investigation using graphing technology, the graphical and algebraic representations of polynomial (i.e., (i.e., linear, quadratic, cubic, quartic) functions * describe key features of the graphs of polynomial functions * distinguish polynomial functions from sinusoidal and exponential functions, and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions * substitute into and evaluate polynomial functions expressed in function notation, including functions arising from real-world applications * pose problems based on real-world applications that can be modelled with polynomial functions, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation; * recognize, using graphs, the limitations of modelling a real-world relationship using a polynomial function, and identify and explain any restrictions on the domain and range * factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring) * make connections, through investigation using graphing technology, between a polynomial function given in factored form and the *x*-intercepts of its graph, and sketch the graph of a polynomial function given in factored form using its key features * determine, through investigation using technology, and describe the connection between the real roots of a polynomial equation and the *x*-intercepts of the graph of the corresponding polynomial function * solve polynomial equations in one variable, of degree no higher than four by selecting and applying strategies (i.e., common factoring; difference of squares; trinomial factoring), and verify solutions using technology * solve problems algebraically that involve polynomial functions and equations of degree no higher than four, including those arising from real-world applications * identify and explain the roles of constants and variables in a given formula expand and simplify polynomial expressions involving more than one variable, including expressions arising from real-world applications * solve equations of the form  using rational exponents * determine the value of a variable of degree no higher than three, using a formula drawn from an application, by first substituting known values and then solving for the variable, and by first isolating the variable and then substituting known values * make connections between formulas and linear, quadratic, and exponential functions, using a variety of tools and strategies * solve multi-step problems requiring formulas arising from real-world applications   **Trigonometric Functions:**   * solve multi-step problems in two and three dimensions, including those that arise from real-world applications * solve problems involving oblique triangles, including those that arise from real-world applications, using the sine law (including the ambiguous case) and the cosine law * make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios or cosine ratios, with or without technology, defining this relationship as the function *f*(*x*) = sin *x* or *f*(*x*) = cos *x*, and explaining why the relationship is a function * determine the values of the trigonometric ratios for angles less than 360º, and solve problems using the primary trigonometric ratios, the sine law, and the cosine law; * make connections between the numeric, graphical, and algebraic representations of sinusoidal functions; * demonstrate an understanding that sinusoidal functions can be used to model some periodic phenomena, and solve related problems, including those arising from real-world applications. * determine, through investigation using technology, the roles of the parameters *d* and *c* in functions of the form *y* = sin (*x* – *d*) + *c* and *y* = cos (*x* – *d*) + *c*, and describe these roles in terms of transformations on the graphs of *f*(*x*) = sin *x* and *f*(*x*) = cos *x* with angles expressed in degrees (i.e., vertical and horizontal translations) * determine, through investigation using technology, the roles of the parameters *a* and *k* in functions of the form *y* = *a* sin *kx* and *y* = *a* cos *kx*, and describe these roles in terms of transformations on the graphs of *f*(*x*) = sin *x* and *f*(*x*) = cos *x* with angles expressed in degrees (i.e., reflections in the axes; vertical and horizontal stretches and compressions to and from the *x*- and *y*-axes) * determine the amplitude, period, and phase shift of sinusoidal functions whose equations are given in the form *f*(*x*) = *a* sin (*k*(*x* – *d*)) + *c* or *f*(*x*) = *a* cos (*k*(*x* – *d*)) + *c*, and sketch graphs of *y* = *a* sin (*k*(*x* – *d*)) + *c* and *y* = *a* cos (*k*(*x* – *d*)) + *c* by applying transformations to the graphs of *f*(*x*) = sin *x* and *f*(*x*) = cos *x* * represent a sinusoidal function with an equation, given its graph or its properties identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range * pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology, in degree mode, from a table of values or from its equation   **Applications of Geometry:**   * represent a vector as a directed line segment, with directions expressed in different ways, and recognize vectors with the same magnitude and direction but different positions as equal vectors * resolve a vector represented as a directed line segment into its vertical and horizontal components * determine, through investigation using a variety of tools and strategies (i.e., head-to-tail method; parallelogram method; resolving vectors into their vertical and horizontal components), the sum (i.e., resultant) or difference of two vectors * solve problems involving the addition and subtraction of vectors, including problems arising from real-world applications * solve problems involving the areas of rectangles, parallelograms, trapezoids, triangles, and circles, and of related composite shapes, in situations arising from real-world applications * solve problems involving the volumes and surface areas of spheres, right prisms, and cylinders, and of related composite figures, in situations arising from real-world applications | **MAP4C Foundations for College Mathematics (College)**  **Mathematical Models:**   * determine, through investigation, the exponent laws for multiplying and dividing algebraic expressions involving exponents and the exponent law for simplifying algebraic expressions involving a power of a power * simplify algebraic expressions containing integer exponents using the laws of exponents * solve simple exponential equations numerically and graphically, with technology, and recognize that the solutions may not be exact * solve problems involving exponential equations arising from real-world applications by using a graph or table of values generated with technology from a given equation * solve exponential equations in one variable by determining a common base * recognize that a linear model corresponds to a constant increase or decrease over equal intervals and that an exponential model corresponds to a constant *percentage* increase or decrease over equal intervals, select a model (i.e., linear, quadratic, exponential) to represent the relationship between numerical data graphically and algebraically, using a variety of tools and strategies, and solve related problems * solve equations of the form using rational exponents * determine the value of a variable of degree no higher than three, using a formula drawn from an application, by first substituting known values and then solving for the variable, and by first isolating the variable and then substituting known values * make connections between formulas and linear, quadratic, and exponential functions, using a variety of tools and strategies * solve multi-step problems requiring formulas arising from real-world applications   **Geometry and Trigonometry:**   * solve problems involving the volumes and surface areas of rectangular prisms, triangular prisms, and cylinders, and of related composite figures, in situations arising from real world applications * solve problems in two dimensions using metric or imperial measurements, including problems that arise from real-world applications, by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios, and of acute triangles using the sine law and the cosine law * solve problems involving oblique triangles, including those that arise from real-world applications, using the sine law (in nonambiguous cases only) and the cosine law, and using metric or imperial units   **Data Management:**   * determine an algebraic summary of the relationship between two variables that appear to be linearly related (i.e., the equation of the line of best fit of the scatter plot), using a variety of tools and strategies, and solve related problems * determine whether a linear model (i.e., a line of best fit) is appropriate given a set of two-variable data, by assessing the correlation between the two variables (i.e., by describing the type of correlation as positive, negative, or none; by describing the strength as strong or weak; by examining the context to determine whether a linear relationship is reasonable) | **MEL4E Mathematics for Work**  **and Everyday Life (Workplace)**  **Applications of Measurement:**   * solve problems involving the areas of rectangles, triangles, and circles, and of related composite shapes, in situations arising from real-world applications * solve problems involving the volumes and surface areas of rectangular prisms, triangular prisms, and cylinders, and of related composite figures, in situations arising from real-world applications |